Sum-to-Product Formulas

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

Let $A = \frac{x+y}{2}$ and $B = \frac{x-y}{2}$

$$A + B = \frac{x+y}{2} + \frac{x-y}{2} = \frac{x+y+x-y}{2} = \frac{2x}{2} = x$$

$$A - B = \frac{x+y}{2} - \frac{x-y}{2} = \frac{x+y-x+y}{2} = \frac{2y}{2} = y$$

$$\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \frac{1}{2}\left[\sin x + \sin y\right]$$

$$2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = \sin x + \sin y$$

$$\boxed{\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}}$$

$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

Replace y with $-y$,

$\sin x + \sin(-y) = 2 \sin \frac{x+(-y)}{2} \cos \frac{x-(-y)}{2}$

$\sin(-y) = -\sin y$

$\boxed{\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}}$

$\boxed{\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}}$

$\boxed{\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}}$

Write $\sin 2x - \sin 7x$ as a product

Use $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

$\sin 2x - \sin 7x = 2 \cos \frac{2x+7x}{2} \sin \frac{2x-7x}{2}$

$= 2 \cos \frac{9x}{2} \sin \frac{-5x}{2}$

Recall $\sin(-A) = -\sin A$

$= \boxed{-2 \cos \frac{9x}{2} \sin \frac{5x}{2}}$

Show $\cos 87^\circ + \cos 33^\circ = \sin 63^\circ$

use $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$$\cos 87^\circ + \cos 33^\circ = 2\cos\frac{87^\circ+33^\circ}{2}\cos\frac{87^\circ-33^\circ}{2}$$

$$= 2\cos\frac{120^\circ}{2}\cos\frac{54^\circ}{2}$$

$\cos A = \sin(90^\circ - A)$

$$= 2\cos 60^\circ \cos 27^\circ$$

$$= 2\cdot\frac{1}{2}\cos 27^\circ$$

$$= \cos 27^\circ = \sin(90^\circ - 27^\circ)$$

$$= \sin 63^\circ \checkmark$$

Find the exact Value of $\cos 255^\circ - \cos 195^\circ$

use $\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$$\cos 255^\circ - \cos 195^\circ = -2\sin\frac{255^\circ+195^\circ}{2}\sin\frac{255^\circ-195^\circ}{2}$$

$$= -2\sin 225^\circ \sin 30^\circ$$

$$= -2(-\sin 45^\circ)\cdot\frac{1}{2}$$

$$= -2\cdot\frac{-\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{2}}{2}}$$

Graph $y = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$

$$y = \frac{\sin 3x \cdot \cos x - \cos 3x \cdot \sin x}{\sin x \cos x} \longrightarrow \sin A \cos B - \cos A \sin B = \sin(A - B)$$

$$= \frac{\sin(3x - x)}{\sin x \cos x} = \frac{\sin 2x}{\sin x \cos x}$$

$$= \frac{2 \sin x \cos x}{\sin x \cos x} \Rightarrow y = 2$$